Week 10 PYQs
Machine Learning Foundations
May 22 - Sep 25

Note

1. If no questions are listed in that term Quiz, it indicates that the corresponding quiz did not include questions from that week.

2. The answer key may contain minor inaccuracies, as answers were taken directly from the original quiz/exam papers. Some of these might have been corrected later.

3. For any issues or suggestions regarding this document, feel free to contact:
   `23f3004032@ds.study.iitm.ac.in`
   or reach out on Discourse: `@23f3004032`

Quiz 1 May 22

Quiz 2 May 22

ET May 22

1. What is the maximum value possible for the function $f(x, y) = e^{2xy}$ subject to the conditions $x \geq 0, y \geq 0$ and $x^2 + y^2 = 2$? *NOTE: Enter your answer upto one decimal.* [Ans: 7.1 to 7.7]

Quiz 1 Sep 22

Quiz 2 Sep 22

ET Sep 22

1. Using Karush-Kuhn-Tucker conditions, solve the following problem. maximize $f(x, y) = xy$ subject to $x + y^2 \leq 2$, $x, y \geq 0$ The global maximum is at

   (a) $(\frac{2}{3}, \frac{4}{3})$

   (b) $(\frac{4}{3}, \sqrt{\frac{1}{2}})$

   (c) $(\frac{2}{3}, \frac{1}{3})$

   (d) $(\frac{4}{3}, \sqrt{\frac{2}{3}})$

[Ans: 4]

Quiz 1 Jan 23

Quiz 2 Jan 23

ET Jan 23

Quiz 1 May 23

Quiz 2 May 23

ET May 23

1. A consumer wants to maximize his utility subject to some constraints. He consumes two goods g_1 and g_2 and the utility function is the sum of g_1 and g_2. His budget is Rs. 100. The per unit price of goods g_1 and g_2 are Rs.3 and Rs.4 respectively. Chose the correct optimization problem.

   (a) maximize $g_1 + g_2$ subject to $3g_1 + 4g_2 \leq 100$

   (b) maximize $g_1 g_2$ subject to $3g_1 + 4g_2 \leq 100$

   (c) maximize $g_1 g_2$ subject to $3g_1 + 4g_2 \geq 100$

   (d) maximize $g_1 + g_2$ subject to $3g_1 + 4g_2 \geq 100$

   [Ans: 1]

2. Consider the following optimization problem involving the variable $x \in \mathbb{R}^2$ $\min_x x_1^2 - 3x_2$ sub. to $x_1 + x_2 \leq 5$ What is the output of the following expression for $x' = [3 \quad 4]^T$ $\max_{\lambda \geq 0} L(x', \lambda)$ Here, $L(x', \lambda)$ is the Lagrangian function for this optimization problem evaluated at x'.

   (a) $(x_1')^2 - 3(x_2')$

   (b) ∞

   (c) $-\infty$

   (d) 0

   [Ans: 2]

Quiz 1 Sep 23

Quiz 2 Sep 23

ET Sep 23

1. A firm produces two products, A and B, with respective manufacturing costs of Rs. 20 and Rs. 30 per unit. The objective is to minimize the total manufacturing cost while meeting certain production constraints. A minimum of 250 units must be produced daily, and at least 1000 machine hours should be utilized each day. Machine hours consumption per unit is 6 hours for A and 5 hours for B. Let the number of units of A produced per day be x_1 and the number of units of B produced per day be x_2. Then, choose the correct optimization problem from the following:

   (a) Minimize: $x_1 + x_2$ Subject to: $x_1 + x_2 \geq 250, 6x_1 + 5x_2 \geq 1000, x_1, x_2 \geq 0$

   (b) Minimize: $20x_1 + 30x_2$ Subject to: $x_1 + x_2 \leq 250, 6x_1 + 5x_2 \leq 1000, x_1, x_2 \geq 0$

   (c) Minimize: $x_1 + x_2$ Subject to: $x_1 + x_2 \geq 250, 6x_1 + 5x_2 \geq 1000, x_1, x_2 \geq 0$

   (d) Minimize: $20x_1 + 30x_2$ Subject to: $x_1 + x_2 \geq 250, 6x_1 + 5x_2 \geq 1000, x_1, x_2 \geq 0$

   [Ans: 4]

Quiz 1 Jan 24

Quiz 2 Jan 24

ET Jan 24

1. A person went to the market to buy hens. Old hens can be bought for 200.00 each but young hens can be bought for 500.00 each. The old hens lay 3 eggs per week and the young hens lay 5 eggs per week. Each egg costs Rs. 11. A hen costs 50.00 per week to feed. If the financial constraint is 8000 per week to buy hens and the capacity constraint is that the total number of hens bought cannot exceed 20 per week. The objective is to earn more profit. Let x represent the number of old hens and y represent the number of young hens. Use the above information to answer the given subquestions.

   Choose the correct Primal optimization problem from the following.

   (a) Maximize: $5y - 17x$
   Subject to: $x + y \leq 20, 200x + 500y \leq 8000, x \geq 0, y \geq 0$

   (b) Maximize: $-5y + 17x$
   Subject to: $x + y \leq 20, 200x + 500y \leq 8000, x \geq 0, y \geq 0$

   (c) Maximize: $33y + 55x$
   Subject to: $x + y \leq 20, 200x + 500y \leq 8000, x \geq 0, y \geq 0$

   (d) Maximize: $55y - 17x$
   Subject to: $x + y \leq 20, 200x + 500y \leq 8000, x \geq 0, y \geq 0$

   [Ans: 1]

2. If (x^*, y^*) is the optimal solution of the primal, u_1 and u_2 are Lagrange multipliers then which of the following options is/are true?

   (a) Complimentary Slackness Condition is $u_1 + 200u_2 = -17, u_1 + 500u_2 = 5$

   (b) Stationary condition is $u_1 + 200u_2 = -17, u_1 + 500u_2 = -5$

   (c) Complimentary Slackness Condition is $u_1(x^* + y^* - 20) = 0, u_2(200x^* + 500y^* - 8000) = 0$

   (d) Complimentary Slackness Condition is $u_1(x^* + y^* - 20) = 0, u_2(200x^* + 500y^* + 8000) = 0$

   (e) Stationary condition is $u_1(x^* + y^* - 20) = 0, u_2(200x^* + 500y^* - 8000) = 0$

   [Ans: 3]

3. Which of the following options is/are true?

   (a) The Given LPP is infeasible.

   (b) The Given LPP is feasible.

   (c) The optimum value is $x = 0$ and $y = 16$

   (d) The optimum value is $x = 20$ and $y = 0$

   [Ans: 2, 3]

Quiz 1 May 24

Quiz 2 May 24

ET May 24

1. Which of the following options is/are true?

   (a) $f(x) = (x^2 - 3x + 2)(x^2 - 7x + 12)$ is a convex function.

   (b) $f(v) = v^T Av$ is a convex function, where $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ and $v = \begin{bmatrix} x \\ y \end{bmatrix}$

   (c) $f(x, y) = x^2 + y^2 + 3$ is a convex function.

   (d) $f(v) = v^T Av$ is a convex function, where $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $v = \begin{bmatrix} x \\ y \end{bmatrix}$.

   [Ans: 2, 3, 4]

Quiz 1 Sep 24

Quiz 2 Sep 24

ET Sep 24

1. Consider the following optimization problem: Minimize $f(x_1, x_2) = x_1^2 + x_2^2$ subject to $g_1(x_1, x_2) = x_1 + x_2 - 1 \leq 0$ $g_2(x_1, x_2) = -x_1 \leq 0$ $x_1, x_2 \geq 0$ Assume that $x = (x_1, x_2)$ is a point that satisfies the KKT conditions. Based on the above data, answer the given subquestions.

   (a) Which of the following is a necessary condition for a point (x_1, x_2) to be optimal, given that the problem satisfies KKT conditions?

   i. $\nabla f(x) = 0$
   ii. $x_1 = x_2 = 2$
   iii. There exist multipliers $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ such that $\nabla f(x) + \lambda_1 \nabla g_1(x) + \lambda_2 \nabla g_2(x) = 0$
   iv. $x_1 = x_2 = 1$

   [Ans: 3]

   (b) If the optimal solution is on the boundary defined by $x_1 + x_2 - 1 = 0$ which KKT condition must hold?

   i. $\lambda_1 = 0$ and $\lambda_2 > 0$
   ii. $\lambda_1 > 0$ and $\lambda_2 \geq 0$
   iii. $\lambda_1 = 0$ and $\lambda_2 = 0$
   iv. $\lambda_1 > 0$ and $\lambda_2 = 0$

   [Ans: 4]

   (c) If the optimal solution is on the boundary defined by $x_1 + x_2 - 1 = 0$ then find the minimum value of $f(x_1, x_2)$. [Ans: 1]

Quiz 1 Jan 25

Quiz 2 Jan 25

ET Jan 25

1. Which of the following options is/are true?

   (a) $\cos x$ is a convex function.

   (b) e^x is a convex function.

   (c) e^{x^2+2} is a convex function.

   (d) If f and g both are convex, then $f \circ g$ and $g \circ f$ are convex functions.

   [Ans: 2, 3]

2. Consider an optimization problem $\min f(x, y, z, w) = x^2 + y^2 + z^2 + w^2 + 1$ subject to constraints: $x + y + z + w = 1$ and $w \leq \delta$. Let the Lagrangian function is, $L(x, y, z, w, \alpha, \beta) = x^2 + y^2 + z^2 + w^2 + 1 + \alpha(1 - x - y - z - w) + \beta(w - \delta)$. Based on the above data, answer the given subquestions.

   (a) Which of the following statements correctly represent the Karush-Kuhn-Tucker (KKT) conditions for the given optimization problem?

      i. $\nabla L = 0$

      ii. $\alpha \geq 0$

      iii. $\beta \geq 0$

      iv. $\beta(w - \delta) \leq 0$

   [Ans: 1, 3]

   (b) Which of the following statements are correct based on the KKT conditions and the solution to the optimization problem?

      i. $x = y = \frac{\alpha}{2}$

      ii. $z = w = \alpha$

      iii. $\frac{3\beta}{8} \geq \frac{1}{4} - \delta$

      iv. $\frac{3\beta}{8} \leq \frac{1}{4} - \delta$

   [Ans: 1, 3]

   (c) If $\delta > \frac{1}{4}$ then find the value of x. [Ans: 0.25]

Quiz 1 May 25

Quiz 2 May 25

ET May 25

1. Consider a set $S = \{(x, y) | -x - 3y \leq -9 \text{ and } -2x - y \leq -6, x, y \geq 0\}$. Use this information to answer the given subquestions.

   (a) Let $A = (x, y)$ be a point in the set S which is closest to the origin. Choose the correct Primal optimization problem from the following.

   i. Minimize: $x^2 - y^2$ Subject to: $-x - 3y \leq -9, -2x - y \leq -6, x, y \geq 0$
   ii. Minimize: $x^2 + y^2$ Subject to: $-x - 3y \leq -9, -2x - y \leq -6, x, y \geq 0$
   iii. Minimize: $x^2 + y^2$ Subject to: $-x - 3y \geq -9, -2x - y \geq -6, x, y \geq 0$
   iv. Minimize: $-x^2 - y^2$ Subject to: $-x - 3y \leq -9, -2x - y \leq -6, x, y \geq 0$

   [Ans: 2]

   (b) Which of the following options is/are true?

   i. The Lagrangian function is $L(x, y) = -x^2 - y^2 + u_1(-x - 3y + 9) + u_2(-2x - y + 6), u_1, u_2 \geq 0$.
   ii. The Lagrangian function is $L(x, y) = x^2 + y^2 + u_1(-x - 3y + 9) + u_2(-2x - y + 6), u_1, u_2 \geq 0$.
   iii. Stationary conditions are $2x - u_1 - 2u_2 = 0$ and $2y - 3u_1 - u_2 = 0$
   iv. Stationary conditions are $2x - 2u_1 - u_2 = 0$ and $2y - 3u_1 - u_2 = 0$.
   v. Stationary conditions are $2x - u_1 - 2u_2 = 0$ and $2y - 3u_1 + u_2 = 0$.

   [Ans: 2, 3]

   (c) If $A = (x, y)$ is the closest point to the origin, then find the value of $3x - y$. [Ans: 7.8]

   (d) Which of the following options is/are true?

   i. S is a convex set.
   ii. The objective function of the Primal optimization problem from the previous question is a convex function.
   iii. The objective function of the Primal optimization problem from the previous question is a concave function.
   iv. None of these

   [Ans: 1, 2]

Quiz 1 Sep 25

Quiz 2 Sep 25

ET Sep 25